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Moreover, Method 2 assisted students to ask more thinking and hinting questions than Method 3. Working cooperatively (high-ability and low-ability students), Method 2 gave an opportunity to the students to discuss, clarify ideas, and evaluate each others' ideas. According to Vygotsky (1978), students are capable of performing at higher levels when working cooperatively than when working individually. Group diversity in terms of knowledge and experience contributes positively to the learning process. Within the cooperative learning environment, the students are confronted with different interpretations of a given situation, and thus, Method 2 created cognitive conflicts among the students which then enhanced them to discuss, explain, evaluate, and modify their opinions to reequilibrate their thinking to learn with understanding. Also Method 2 provided the students with opportunities to learn from each other's skills and experiences.

It can be concluded that embedding metacognitive scaffolding within cooperative learning enhanced students' mathematical reasoning and metacognitive knowledge. When students are actively engaged in activities such as planning, monitoring, questioning, explaining, elaborating, negotiating meanings, constructing arguments, and evaluation, they benefit much from the cooperative learning process. Thus, Method1 is superior to Method 2 alone. In other words, to maximize the benefits of Method 2 in enhancing mathematical reasoning and metacognitive knowledge, the cooperative learning process should be scaffolded appropriately, and modeling through metacognitive scaffolding.

### **Recommendations and Suggestions**

From the discussion of the findings, it is evident in this study that Method1 is effective in supporting students' mathematical reasoning and metacognitive knowledge. Thus, metacognitive scaffolding can be integrated in instructional design, curriculum design, computer based design, or web-based design. Additionally, Method1 should be included in teacher education programs. There are several skills, such as grouping, drawing metacognitive questions, and reflection, that pre-service and in-service teachers need to be trained. Also the use of Method1 in the classroom requires an approach to assessment and evaluation that is different from the present system. A more authentic and performance-based assessment criteria, that pre-service and in-service teacher need to be trained to develop to accompany the implementation of this method in the classroom. Finally, the implementation of Method1 is not costly. Therefore, the cost effectiveness of this method make this method a good candidate for inclusion in the development of the pedagogical approach.

An interesting question raised in this study relates to the effects of embedding metacognitive scaffolding within cooperative setting versus embedding metacognitive scaffolding within individual learning on mathematical reasoning and metacognitive knowledge. To address the issue, students who worked cooperatively and used metacognitive questions cards should be compared with students working individually and using metacognitive questions cards. The researcher suggests further research in teachers' and students' attitudes toward Method1. Also, different subjects with different stages are worth to be investigated in future research.

The present study was limited to the "Adding and Subtracting Fractions" unit in the male fifth-grade textbooks, and this may restrict generalizing the study findings to the rest of mathematics concepts and subjects and may restrict generalizing the findings to the females.

**Note.** The adjusted mean differences shown in this table are the subtraction of the second condition (on the lower line) from the first condition (on the upper line); for example, 1.97 (Adjusted Mean Difference for Mathematical Reasoning) = **Method1 – Method2**.

## Discussion and Conclusions

The present study investigated the effects of metacognitive scaffolding embedded in cooperative setting method (Method1) and cooperative learning alone method (Method2) on students' mathematical reasoning and metacognitive knowledge. The findings indicated that students taught via Method1 significantly outperformed their counterparts taught via Method2 and via Method 3 in mathematical reasoning and metacognitive knowledge. Also, students taught via Method 2 significantly outperformed students taught via the traditional method (Method3) in mathematical reasoning and metacognitive knowledge.

The effectiveness of Method1 on mathematical reasoning support other findings (Brown and Palincsar, 1989; Chi, et al., 1994; Kramarski, et al., 2001, 2002; Lin, et al., 1999; Palincsar, et al., 1987; Slavin, 1996; and Webb, 1982, 1989b) which show that metacognitive strategies are one of the best means of elaborating information and of making connections. By understanding why and how a certain solution to a task and a problem has been reached, the students elaborated on the information gained from the metacognitive questions and learned from it, which in turn, affected mathematical reasoning and students' ability to transfer their knowledge to solve mathematical tasks and problems.

The process of solving tasks at a high level of cognitive complexity (e.g., mathematical reasoning problems) depends on the activation of metacognitive processes more than on solving tasks at a lower level of cognitive complexity because the former requires careful planning, monitoring, regulation, and evaluation (Stein et al., 1996). Method1 enhanced students to activate such processes, so they could reason mathematically better than the students taught via Method2 that focused only on working cooperatively and the students' interaction was not structured. Specifically, the use of metacognitive questions guided students to analyze the entire situation described in the task or in the problem and thereby did not only enhance their understanding, but also enabled them to replace their earlier inappropriate strategies with a new virtually errorless process which is an essential element of mathematical reasoning.

One of the most important components of mathematical reasoning is the appropriate strategies selection and the justification of selecting these strategies. The students taught via Method1, were able to select and justify the appropriate strategies for

solving the problem because they were trained how to do so.

The findings of this study suggest that there were certain conditions in which the use of cooperative learning fully worked to facilitate learning and particularly to enhance mathematical reasoning. Webb (1989b) found that students who learned most were those who provided explanations to others in their group. In this regard, metacognitive questions served to facilitate the cooperative learning processes through eliciting responses from some students, and the responses may invoke further questions from other students who may require elaboration, reasoning, or explanation from their peers. In this study the cooperative learning of the students taught via Method1 was structured and guided by the metacognitive questions cards and therefore these students were assisted to explain and reason their solution processes.

The students taught via Method1 were scaffolded through the cooperation i.e., high-ability and low-ability interaction, and through the use of metacognitive questions which helped the students in narrowing their Zone of Proximal Development (ZPD). The students taught via Method2 were scaffolded through only the cooperation which enabled them to reason mathematically better than students taught via the traditional method (Method 3) whose learning was not scaffolded. Therefore, when the metacognitive scaffolding was provided to students for group cooperation, the benefits of cooperative learning in mathematical reasoning were maximized.

The effectiveness of Method1 on metacognitive knowledge confirms the results of previous studies (e.g., Choi et al., 2005; Davis & Linn, 2000; King, 1991a; Kramarski & Mizrachi, 2006; Lin & Lehman, 1999; Mevarech & Fridkin, 2006; Palincsar & Brown, 1984), which were all consistent in concluding that cooperative learning and questioning strategies enhanced metacognitive knowledge and reflective thinking. The use of metacognitive questions directed students' attention to plan, monitor, and evaluate their learning processes, which helped them to obtain metacognitive knowledge and transfer their understanding to novel problems and situations. The findings of this study support the findings of Chi et al. (1989) and Webb et al. (2009) that metacognitive questions and self-explanation assisted students to make arguments for their solutions and decisions, and thus make thinking explicit.

The findings of this study showed positive effects of Method 2 on mathematical reasoning and metacognitive knowledge. The students taught via Method2 worked cooperatively where multiple responses were provided. This learning environment somewhat encouraged students to produce high level thinking questions and provide evidence for their solutions more than the students taught via Method 3.

**Findings**

Table 1 presents overall means, standard deviations, adjusted means, and standard errors of each dependent variable by the instructional method.

**Table 1:** Means, standard deviations, adjusted means and standard errors for each dependent variable by the instructional method

Dependent Variables	The Instructional Method			
	Method1 N= 80	Method2 N= 79	Method3 N= 81	
<b>Mathematical Reasoning (MR)</b>	Mean	16.15	14.16	12.72
	SD	2.3	2.7	2.4
	Adj. mean	16.25 <sup>a</sup>	14.28 <sup>a</sup>	12.51 <sup>a</sup>
	Std. Error	.168	.169	.167
<b>Metacognitive Knowledge (MK)</b>	Mean	2.29	1.94	1.72
	SD	.25	.30	.27
	Adj. mean	2.30 <sup>a</sup>	1.95 <sup>a</sup>	1.70 <sup>a</sup>
	Std. Error	.024	.025	.024

Note. a. Evaluated at covariates appeared in the model: pre-MR = 5.5250.

Total score on MR = 22 and total score on MK is out of 3.

To examine if there were statistically significant differences in MR and MK adjusted mean scores between Method1, Method2, Method3, while controlling the pre-MR, one-way multivariate analysis of covariance (MANCOVA) was conducted (run on SPSS). Table 2 presents the results of MANCOVA. The results indicated statistical significant differences,  $F(2,237) = 55.86, p = .000$ . The covariates pre-MR  $F(2,237), p = .000$  had statistical significant effects. Further, the results of the univariate ANCOVA tests, which are represented in table 2, indicated that there were statistically significant differences in MR and MK. The  $F$  ratio of MR (2, 237) was 124.875,  $p = .000$ . This means that the instructional method had a main effect on MR. This effect accounted for 51% of the variance of MR ( $\text{Eta}^2 = .514$ ). The  $F$  ratio of MK (2, 237) was 153.254,  $p = .000$ . This means that the instructional method had a main effect on MK. This effect accounted for 57% of the variance of MK ( $\text{Eta}^2 = .565$ ).

**Table2:** Summary of multivariate analysis of covariance (MANCOVA) results by the instructional method and follow-up analysis of covariance (ANCOVA) results.

MANCOVA Dependent and Covariates	Effect, Variables, Pillai's Trace	Multivariate F	Univariate F df = 2, 237
<b>Group Effect</b>		55.86 ( $p = .000$ )	-
<b>Mathematical Reasoning(MR)</b>		-	124.875 ( $p = .000$ )
<b>Metacognitive Knowledge (MK)</b>		-	153.254 ( $p = .000$ )
<b>Pre-MR</b>		223.06 ( $p = .000$ )	-

To identify significantly where the differences in the adjusted means resided, a post hoc pairwise comparison using the /lmatrix command was conducted (run on SPSS). Table 3 is a summary of post hoc pairwise comparisons. The post hoc pairwise comparison results showed that students in Method1 significantly outperformed students in Method2 and in Method 3 in mathematical reasoning and metacognitive knowledge. The results also showed that students in Method 2 significantly outperformed students in Method 3 in mathematical reasoning and metacognitive knowledge. The adjusted mean differences are presented below.

*Mathematical Reasoning.* Students in Method1 ( $M = 16.15, SD = 2.3, \text{Adj.m} = 16.25$ ) significantly outperformed students in Method2 and Method3, with an adjusted mean difference of 1.97 ( $F = 68.62, p = .000$ ) and 3.74 ( $F = 249.53, p = .000$ ) respectively. On other hand, students in Method2 ( $M = 14.16, SD = 2.7, \text{Adj.m} = 14.28$ ) significantly outperformed the students in Method3 (Mean = 12.72,  $SD = 2.4, \text{Adj.mean} = 12.51$ ) with an adjusted mean difference of 1.77 ( $F = 55.56, p = .000$ ). (Effect sizes on MR were .83 and .60 for comparing Method1 and Method2, and Method2 and Method3, respectively).

*Metacognitive Knowledge.* Students in Method1 ( $M = 2.29, SD = .25, \text{Adj.m} = 2.30$ ) significantly outperformed students in Method2 and Method3, with an adjusted mean difference of (.35,  $F = 100.54, p = .000$  and .60,  $F = 304.15, p = .000$ ) respectively. On the other hand, students in Method2 ( $M = 1.94, SD = .30, \text{Adj.m} = 1.95$ ) significantly outperformed students in Method3 ( $M = 1.72, SD = .27, \text{Adj.m} = 1.70$ ) with an adjusted mean difference of (.25,  $F = 53.99, p = .000$ ) (Effect sizes on MK were 1.29 and .81 for comparing Method1 and Method2, and Method2 and Method3, respectively).

**Table 3:** Summary of post hoc pairwise comparisons

Comparison Group	Dependent Variables			
	<u>Mathematical Reasoning (MR)</u> Adj.Mean Difference	Sig	<u>Metacognitive Knowledge (MK)</u> Adj.Mean Difference	Sig
<b>Method1 vs. Method2</b>	1.97	.000	.35	.000
<b>Method1 vs. Method3</b>	3.74	.000	.60	.000
<b>Method2 vs. Method3</b>	1.77	.000	.25	.000

mentioned about numerator or transferring it into a common denominator). A score of 1 indicates an explanation that has some satisfactory elements but may have omitted a significant part of the task (e.g.,  $\frac{9}{5} > \frac{2}{3}$  because when transformed into a common denominator the numerator 27 is bigger than the numerator 10. Nothing is mentioned about the denominators. A score of 2 indicates a clear, unambiguous explanation of student's mathematical reasoning (e.g.,  $\frac{9}{5} > \frac{2}{3}$ , when transform into equivalent fractions with a like denominator  $\frac{27}{15}$  and  $\frac{10}{15}$ , the fraction with the larger numerator is the larger fraction if the denominators are the same, since the denominators (15, 15) are same and the numerator 27 is bigger than the numerator

$$10, \frac{27}{15} > \frac{10}{15}.$$

The real-life problem: A scoring rubrics was adapted from the Kramarski et al. (2001) procedure with a repeated .86 interjudge reliability. Two criteria, which tightly correspond to the mathematical reasoning were identified. Students' answers were scored on these criteria, each criterion ranges from 0 (no solution) to 3 (highest level solution), and a total score ranging from 0 to 6. The criteria were:

- 1- Referencing all data (referring to all data in each of the two offers: mixed fruit juice volume, components, and prices. Identifying relationships, distinguishing relevant from irrelevant information).
- 2- Making justifications for the suggested solution (giving reasons, providing evidence, and justifying the suggestion).

To measure students' metacognitive knowledge, a metacognitive knowledge questionnaire was used. The metacognitive knowledge questionnaire was adapted from the study of Montague and Bos (1990), assessed students' metacognitive knowledge regarding their problem-solving strategies, and from Xun (2001) self-report questionnaire. The metacognitive knowledge questionnaire of this study consisted of 15 items grouped into three categories. The first category (5 items) focused on strategies used before the solution process (planning); the second (5 items) category focused on strategies used during the solution process (monitoring); and the third (5 items) focused on strategies used at the end of the solution process (evaluation). Each item was constructed of a 3-point, Likert-type scale ranging from 1 (never) to 3 (always) and a total mean score ranging from 1 to 3.

### Instruments' Validity

Content validity of the the mathematical reasoning test items ,the metacognitive knowledge questionnaire

items, the scoring procedure of mathematical reasoning items, and the scoring rubrics of assessing the real-life problem were assessed by two experienced mathematics teachers, two education mathematics supervisors, and two mathematics education university lecturers in Jordan. The evaluators' suggestions, feedback, and comments were taken into account until there were no discrepancies among the evaluators.

### Instruments' Reliability

A pilot study was conducted in order to test the instruments' reliability. 80 students were randomly selected from a randomly selected primary school, who were not going to participate in the formal study .The mathematical reasoning test and the metacognitive knowledge questionnaire were carried out and the scores were collected to determine the Cronbach's Alpha reliability coefficient. Cronbach's alpha reliability coefficient of the mathematical reasoning test was .88 and it was .84 for the metacognitive knowledge questionnaire. Cronbach's Alpha reliability coefficients for the metacognitive questionnaire categories were .64, .66, .60 for planning, monitoring, and evaluation respectively. The Cronbach's Alpha reliability coefficients showed that the study instruments were satisfactory and reliable.

To control any possible differences between the students' ability in the three groups, two months before implementing the study, all students in the three conditions were given the pre-test. Within each school, the teachers continued conducting classes according to their assigned teaching methods until the end of the first semester. In the present study, the focus was on the "Adding and Subtracting Fractions" unit that was taught in all classrooms for 14 sessions. At the end of implementing the study, all students were asked to administer the post test.

### Techniques of Data Analysis

While there was a significant correlation (.76 significant at the .01 level) among the dependent variables (MR and MK), and there was an independent variable with three levels and two dependent variables, and pre-test as a covariate, one-way multivariate analysis of covariance (one-way MANCOVA) was conducted to compare the three adjusted mean scores on MR and MK. Because the overall one-way MANCOVA results were statistically significant, a series follow up one-way analysis of covariance (one-way ANCOVAs) were used to identify where the differences resided. Since the follow up ANCOVAs results were statistically significant, the post hoc pair wise comparison technique using the /matrix command was used to identify where the differences in adjusted means resided. All of the statistical analysis tests were computed at 0.05 level of significance.

scaffolding input by the teacher was gradually reduced, for example, the teacher's time in the first session, was 30 minutes; in the second session it was about 25 minutes; in the third session, it was about 20 minute and so on until the time became when the teacher taught for about 10 minutes regarding the new topic and the students continued learning by their own using the metacognitive questions cards.

## 2-The cooperative learning group (Method2)

In this group, students were trained and guided to work cooperatively. In the first session, the teacher introduced and explained the new topic for 25 minutes to the whole class and proceeded to teach in a usual manner. For example, he used the board and explained the main ideas of today's lesson. After the teacher's explanation of the new topic to the whole class, students were asked to do their exercises and solve the assigned mathematical problems in groups for 15 minutes. The reader read the problem aloud; the colleagues discussed the learning task and asked themselves different questions. The summarizer, the recorder, and the presenter played the same roles of their counterparts in Method 1. During the session, the teacher intervened when needed to improve task work and teamwork. At the end of the session, the teacher evaluated students' performance and had students celebrate the work of group members. For the next sessions, the teacher and students followed the same method and procedures. However, group members' roles were rotated each session.

## 3-The control group (Method 3).

The control group served as a comparison group with no intervention. Therefore, the teacher of this group continued teaching as he usually did.

## Monitoring the Implementation of the Study

During the first two months of implementing this study, three mathematics education supervisors, whose job was to regularly visit the three teachers in their classes, visited the three teachers twice a month. Each mathematics education supervisor was informed to observe his assigned teacher following the checklists prepared by the researcher to ensure the fidelity to the implementation. The checklist of Method 1 contained questions such as: Did the teacher ask metacognitive questions during his explanations? Did the teacher assign the groups correctly? Did the teacher gradually reduce his metacognitive scaffolding input? Did the teacher distribute the metacognitive questions cards to the all groups? Did each group member play different roles? The checklist of Method 2 contained questions such as: Did the teacher assign the groups correctly? How long did the teacher's explanation last? How long did the students work cooperatively? Did each group member play different roles? The checklist of Method 3

contained questions such as: How long did the teacher's explanations last? How long did student spend to solve the mathematics problems individually?

During the last month of implementing this study, namely, during the teaching of "Adding and Subtracting Fractions Unit", the three mathematics education supervisors visited the three teachers twice a week and followed the same checklists to ensure the implementation fidelity. Also, the researcher met each teacher twice a week to ensure fidelity to the treatment following the checklists used by the three mathematics education supervisors.

## Instruments

To measure students' mathematical reasoning, a pre- test and post-test were used in this study. The mathematical reasoning pre-test and post-test questions were similar in content but their order and numbering were randomized. Based on the learning objectives, a specifications table was constructed. The specifications table contained (4) dimensions: using estimation to verify the reasonableness of calculated results (Q 8), explanation (Q1, Q2,Q4), support solutions with evidence (Q3, Q5), and providing reasonable justifications for the solution (Q6 ,Q 7).

The mathematical reasoning test consisted of 8 items and a real-life problem that was developed by the researcher . The test covered the following topics: equivalent fractions, simplifying fractions, comparing and ordering fractions and mixed numbers, adding and subtracting fractions, and adding and subtracting mixed numbers. The test items were composed of two kinds of items. One kind (8 items) was based on open-ended tasks. For example:

$\frac{9}{5} \dots \frac{2}{3}$ , explain which sign  $>$ ,  $<$ , or  $=$  will make the statement true.

The second kind was a real-life problem. The problem asked students to decide the better buy from two different prices and quality of mixed fruit juice. The student had to decide the better buy.

The total score of the test was 22 (16 for the open-ended tasks items and 6 for the real-life problem). The 8 mathematical reasoning test items and the real-life problem scoring were as follows:

Open-ended task items: For each item, students received a score between 0 and 2, and a total score ranging from 0 to 16. For example, "In the following item,  $\frac{9}{5} \dots \frac{2}{3}$ , explain which sign  $>$ ,  $<$ , or  $=$  that will make the statement true." A score of 0 indicates incorrect selection and explanations or explanations that are irrelevant to the task (e.g.,  $\frac{9}{5} < \frac{2}{3}$  because when the denominator is smaller the value is greater. Nothing is



cooperative setting group, 79 students in the cooperative learning alone group, and 81 students in the traditional group. Each of the three male teachers who participated in this study taught his assigned group in his school. All the teachers were males who had similar levels of education (B.Ed. major in mathematics), had more than 7 years of experience in teaching mathematics.

### Study Design

This quasi-experimental study was designed to investigate the effects of metacognitive scaffolding embedded in cooperative setting and cooperative learning methods on mathematical reasoning and metacognitive knowledge. The study employed (Pre-test – Treatment – Post-test) design by comparing three groups while using the pre-test as a covariate. It was designed to investigate the effects of the independent variable on the dependent variables while controlling students' pre-levels. The research design is illustrated as the followings:

O<sub>1</sub> X<sub>1</sub> Y<sub>1</sub> O<sub>2</sub> X<sub>1</sub>: Method 1 O<sub>1</sub> = O<sub>3</sub> = O<sub>5</sub> = Pre-test  
 O<sub>3</sub> X<sub>2</sub> Y<sub>1</sub> O<sub>4</sub> X<sub>2</sub>: Method 2 O<sub>2</sub> = O<sub>4</sub> = O<sub>6</sub> = Post-test  
 O<sub>5</sub> X<sub>0</sub> Y<sub>1</sub> O<sub>6</sub> X<sub>0</sub>: Method 3

The independent variable of this study was the instructional method with three categories:

1. Metacognitive scaffolding embedded in cooperative setting method (Method 1).
2. Cooperative learning method (Method 2).
3. Traditional instructional method (Method 3).

The dependent variables were:

1. Mathematical reasoning (MR).
2. Metacognitive knowledge (MK).

### Teachers' Training

The teachers who taught the metacognitive scaffolding embedded in cooperative setting method and cooperative learning alone groups were exposed to one week training on the instructional methods. The metacognitive scaffolding embedded in cooperative setting method's teacher was exposed to some examples about the nature of the metacognitive questions and how to use and train students to use the metacognitive questions cards in a cooperative learning setting. He was informed to use metacognitive questions in his explanations and coach his students to use metacognitive questions when they solve the mathematical problems. The procedures of selecting groups and assigning group members were explained to the teacher. The cooperative learning alone method's teacher was trained about teaching mathematics within cooperative learning setting, and about selecting groups and assigning groups' members. Finally, the traditional method's teacher was not exposed to the metacognitive scaffolding or to the cooperative learning training, he was asked to teach as he used to teach in a usual manner. The materials included the mathematics

textbooks, explicit lesson plans, and examples of metacognitive questions.

### Experimental Conditions

In the present study, the focus was on the "Adding and Subtracting Fractions Unit" that was taught in the three groups for 15 sessions (14 sessions for implementing each method and 1 session for administrating the test and questionnaire) within 45 minutes for each. At the end of implementing the study (15th session), all students were asked to answer the mathematical reasoning test questions. After completing the test, they were immediately asked to complete the metacognitive questionnaire. In Method 1 and Method 2, students were assigned into groups based on their ability. They were divided into high and low-abilities based on their pre-test scores in mathematical reasoning. Each group was formed by randomly choosing two high-ability students and two low-ability students. Roles of students were also assigned by the teachers. The teacher and learners applied Method 1 and Method 2 two months before the formal experiment with practice units. The three groups were different from one another as follows:

#### 1- The metacognitive scaffolding embedded in cooperative setting group (Method 1)

In this group students' learning was supported by the metacognitive questions asked by the teacher and by the metacognitive questions cards that used during the cooperative setting. In the first session, the teacher explained the new topic for about 30 minutes to the whole class by asking himself and training students to ask metacognitive questions regarding planning, monitoring, and evaluation. After the teacher's explanation, students worked cooperatively using the metacognitive questions cards that guide and support students to ask metacognitive questions.

In this way, one of the group's members read the problem and asked his colleagues aloud. The colleagues listened to the question and tried to answer. Whenever there was no consensus, the group members discussed the issue until the disagreement was resolved. When the disagreement was resolved, a student orally summarized the solution, the explanation, and the justification and discussed with his colleagues. With the solution, explanation, and justification were in hand, the recorder wrote them down and the presenter presented them to the whole class. During these processes, the teacher monitored each learning group and intervened by asking more metacognitive questions if necessary. At the end of the session, the teacher discussed with the whole class to ensure that students carefully process the effectiveness of their learning group. For the next sessions, the teacher and students followed the same procedures and the group members' roles were rotated after each session. However, the metacognitive

Chi, Leeuw, Chiu, and Lavancher (1989); Lin and Lehman (1999); and Masui and De Corte (1999) found that metacognitive questions facilitated problem-solving processes, assisted students to make arguments for their solutions and decisions, and enabled students to increase their knowledge about orienting and self-judging themselves. Also, Davis and Linn (2000); King (1991a); Kramarski, Mevarech, and Arami (2002); Lin and Lehman (1999); Palincsar and Brown (1984, 1989); and Wineburg (1998) found that when students were trained to ask questions before, during, and after the learning task, they were able to solve a real-life problem in the absence of domain knowledge and their metacognitive knowledge was enhanced.

However, metacognitive strategies according to Piaget's (1970) cognitive development stages require abstract thinking that students become proficient in when they reach the formal operation stage (12 years and above). Young students, for example, 10 year olds need to be supported, guided, or pushed to be metacognitive thinkers.

Vygotsky (1978) indicates that an active student and an active social environment cooperate to produce developmental change. The student actively explores and tries alternatives with the assistance and guidance of a more skilled partner, as in an instructor, or a more capable peer. In this regard, Vygotsky (1978) mentions that there is a hypothetical region where learning and development best take place. He identifies this region as the zone of proximal development (ZPD). This zone is defined as the distance between what an individual can accomplish during independent problem solving, versus what can be accomplished with the help of an adult or a more capable member of a group. Panitz (2009) indicates that teacher and partner support students' activity, scaffolding their efforts to increase current skills and knowledge to a higher competency level. Scaffolding is the support during a teaching session, where a more skilled partner (adult or peer) adjusts the level of assistance given based on the level of performance indicated by the student. Researchers (Choi, Land, & Turjeon 2005; Kramarski & Mizrachi 2006; Mevarech & Fridkin 2006 ) found that scaffolding is an essential instructional element to facilitate learning. Webb et al. (2009) found that when learners were supported to explain their thinking, it helped them to clarify their explanations, justify their reasoning and problem-solving strategies, and correct any misconceptions.

Thus, scaffolding cooperatively and metacognitively, should be provided to support both cognition and metacognition. Cognition refers to domain-specific knowledge and strategies for information and problem manipulation (Schraw,1998), and metacognition includes knowledge of cognition and regulation of cognition (Pintrich Wolters, & Baxter, 2000), such as planning, monitoring, and evaluation.

The two constructs are interrelated. Although metacognitive knowledge may be able to compensate for absence of relevant domain knowledge, its development may also depend on having some relevant knowledge of the domain (Pifarre' and Cobos, 2009).

### **Purpose of the study**

To date, however, research has provided relatively little insight into the role of metacognitive scaffolding on young learners' mathematical reasoning and metacognitive knowledge. While various research studies have been conducted on the separate effects of metacognitive strategies or cooperative learning on mathematics achievement, attitudes, and self-efficacy, no study was found that addresses the effects of metacognitive scaffolding embedded in cooperative setting on mathematical reasoning and metacognitive knowledge.

Therefore, the purpose of this study was to find out the extent to which the metacognitive scaffolding embedded in cooperative setting method could play an important role in enhancing Jordanian fifth-grade students' mathematical reasoning and metacognitive knowledge. Particularly, the study was conducted to investigate if there were any significant differences in mathematical reasoning (MR) and metacognitive knowledge (MK) levels between students taught via the metacognitive scaffolding embedded in cooperative setting method (Method1), students taught via the cooperative learning alone method (Method 2), and students taught via the traditional instructional method (Method 3). As such, the study was focused on the following questions:

- 1- Would students taught via Method 1 perform higher than students taught via Method 2 and students taught via Method 3 in MR and MK?
- 2- Would students taught via Method 2 perform higher than students taught via Method 3 in MR and MK?

### **Method**

#### **Population and Participants**

The population of this study comprised of all male fifth grade students enrolled in the first public educational directorate in Irbid District in Jordan which includes 44 male primary schools.

The participants of this study were 240 fifth grade male students. The mean age of the students was 10.6 years. Three primary schools from the first public educational directorate in Irbid Governorate were randomly selected to participate in this study from a total of 44 male primary schools. To implement this study in a naturalistic school setting, existing intact classes were used. Two classes from each participating school were randomly assigned to each condition 80 students in the metacognitive scaffolding embedded in

National Council of Teachers of Mathematics (2008) and Kramarski (2000) have suggested cooperative learning strategy to enhance learning mathematics with understanding. Johnson and Johnson (2007, p. 404) defined cooperative learning broadly as: “students working together to achieve learning goals.” Researchers agree that mathematical communication within the learning community is crucial for the development of students' mathematical understanding (Alrø and Skovsmose, 2003; Forman, 2003). Vygotsky (1978) affirms that when students interact with each other, they typically will learn, receive feedback, and be informed of something that contradicts with their beliefs or current understanding. This conflict often causes students to recognize and reconstruct their existing knowledge (Rogoff, 1990).

Cooperative learning has been strongly recommended to be used in improving students' cognitive performance, social relationships, and attitudes (Dansereau, 1988; Paris and Winograd, 1990; Tarım, 2009; Tarım, 2003; Tarım & Artut, 2004; Weinstein, Meyer, & Stone, 1994). The report of the National Governors' Association (Brown & Goren, 1993) indicated that within cooperative learning setting, mixed ability students work together to solve problems and complete tasks. In this setting, low-ability students have the opportunity to model the study skills and work habits of more proficient students. In the process of explaining the material, high-ability students often develop greater mastery themselves by developing a deeper understanding of the task.

Johnson, Johnson, and Stanne (1986) found that about 600 experimental and over 100 correlational studies have been conducted since 1898 which have compared competitive, individualistic, and cooperative learning. The majority of the studies showed that cooperative learning has advantages over the competitive and individualistic learning. Researchers (e.g., Acar & Tarhan 2008; Doymus 2008; Johnson, Johnson, & Smith, 1991, 1998; Johnson & Johnson 1999, 2004; Kirschner, Strijbos, Kreijns, & Beers, 2004b; Slavin 1990, 1994, 1991, 1995, 1996; Slavin, Hurley, & Chamberlain, 2003) found that cooperative learning, as opposed to individual learning, promotes greater problem-solving and critical thinking abilities, facilitates retention, and higher reasoning.

However, researchers (Greene & Land 2000; Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2003; Slavin et al. 2003) found that cooperative learning was useful in influencing the development of ideas only when group members offered suggestions, when they were open to negotiation of ideas, and when they shared prior experiences. There may be times when group members do not know how to ask questions or how to elaborate thoughts, or there may be times when group members are not willing to ask questions or respond to others' questions, or there may be times when group

members do not see the need for cooperation. Webb and Mastergeorge's (2003a, b) model of cooperative learning further revealed that different conditions and patterns of cooperation might lead to different learning outcomes. Kramarski and Mizrachi (2006) indicated that students' cooperation needs to be structured and guided to be useful and effective.

One way of structuring and guiding students' cooperation is through the provision of metacognitive strategies. Tarja, Tuire, and Sanna (2006) found a positive relationship between metacognitive strategies and the features of interaction. Metacognitive strategies support students to manage their thinking, recognize when they do not understand something, and adjust their thinking accordingly, not just guide them to master mathematical procedures. In other words, these strategies guide and support students to think before, during, and after a problem solution, and then achieve their learning objectives more effectively. Desoete, Roeyers, & Huylebroeck (2006) found that learners who applied metacognitive strategies achieved significant gains in trained metacognitive skills and mathematical facts retrieval. Also, Cossey (1997) found that the more often students are exposed to metacognitive strategies, the greater their gains are on mathematical reasoning. Metacognitive strategies begin by guiding students to work cooperatively to plan for selecting the appropriate method to accomplish the task, and then continue as they select the most effective method and afterward evaluate their learning process and outcomes. Hoek, Eden, and Terwel (1999) and Mevarech (1999) studies showed that metacognitive strategies are effective for developing the selection of the appropriate methods for problem-solving.

Kramarski, Mevarech, and Lieberman (2001) found that metacognitive strategies helped students to ask themselves questions before (through planning), during (through monitoring), and after (through evaluation) the learning task. For example, at the planning stage a student asks himself or herself metacognitive questions such as: “What in my prior knowledge will help me with this particular task? What should I do first? Do I know where I can go to get some information on this topic? How much time will I need to learn this? What are some strategies that I can use to learn this?” At the monitoring stage a student asks himself or herself metacognitive questions such as: “Did I understand what I just heard, read or saw? Am I on the right track? How can I spot an error if I make one? How should I revise my plan if it is not working? Am I keeping good notes or records?” And at the evaluation stage a student asks himself or herself metacognitive questions such as: “Did my particular strategy produce what I had expected? What could I have done differently? How might I apply this line of thinking to other problems?”

## Effects of Metacognitive Scaffolding Embedded in Cooperative Setting on Mathematical Reasoning and Metacognitive Knowledge

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**Abstract:** The aim of this study was to investigate the effects of metacognitive scaffolding embedded in cooperative setting method (Method1) and cooperative learning alone method (Method2) on fifth-graders' mathematical reasoning and metacognitive knowledge. 240 fifth grade male students were randomly selected from three different schools to participate in this study. Students were divided into three groups: Students in the first group were taught via Method 1, students in the second group were taught via Method2, and students in the third group were taught via Method 3 as a control group. To achieve the aim of the study, Method1 was compared to Method2 and to Method3 using a quasi-experimental design, also Method2 compared to Method3. A mathematical reasoning test and a metacognitive knowledge questionnaire were administered following the implementation of the instructional methods on the unit of "Adding and Subtracting Fractions". Data analyses were carried out using Multivariate Analysis of Covariance (MANCOVA). The results showed that students taught via Method1 significantly outperformed students taught via Method2 and Method3 in mathematical reasoning and metacognitive knowledge. The results also showed that students taught via Method2 significantly outperformed students taught via Method3 in mathematical reasoning and metacognitive knowledge. (Keywords: Metacognitive Scaffolding, Cooperative Learning, Mathematical Reasoning, Metacognitive Knowledge).

### أثر طريقة دعم تعلم مهارات ما وراء المعرفة المضمن في الموقف التعاوني على الاستدلال الرياضي والوعي بتطبيق مهارات ما وراء المعرفة

إبراهيم الجبيلي: قسم تكنولوجيا التعليم، كليات الشرق العربي للدراسات العليا، الرياض، المملكة العربية السعودية

ملخص: هدفت هذه الدراسة إلى استقصاء أثر طريقة دعم تعلم مهارات ما وراء المعرفة المضمن في الموقف التعاوني (الطريقة الأولى)، و طريقة التعلم التعاوني وحده (الطريقة الثانية) على الاستدلال الرياضي والوعي بتطبيق مهارات ما وراء المعرفة لدى طلبة الصف الخامس الأساسي. تم اختيار 240 طالباً عشوائياً من بين ثلاث مدارس مختلفة، وتم تقسيمهم إلى ثلاث مجموعات: المجموعة الأولى تم تدريسها بواسطة طريقة التدريس الأولى، والمجموعة الثانية بواسطة طريقة التدريس الثانية، و المجموعة الثالثة بواسطة طريقة التدريس الثالثة كمجموعة ضابطة. ولتحقيق أهداف هذه الدراسة فقد تم إجراء مقارنة بين كل من طلاب المجموعة الأولى و طلاب المجموعة الثانية و طلاب المجموعة الثالثة من جهة، و مقارنة طلاب المجموعة الثانية و طلاب المجموعة الثالثة من جهة أخرى فيما يتعلق بالاستدلال الرياضي و بالوعي بتطبيق مهارات ما وراء المعرفة. بعد تطبيق الدراسة، تم تطبيق اختبار الاستدلال الرياضي و استبانة الوعي بتطبيق مهارات ما وراء المعرفة في وحدة "جمع و طرح الكسور" على جميع الطلبة في المجموعات الثلاث. و قد أشارت النتائج إلى أن طلاب المجموعة الأولى تفوقوا على طلاب المجموعة الثانية و طلاب المجموعة الثالثة في الاستدلال الرياضي و الوعي بتطبيق مهارات ما وراء المعرفة، كما أشارت النتائج إلى تفوق طلاب المجموعة الثانية على طلاب المجموعة الثالثة في الاستدلال الرياضي و الوعي بتطبيق مهارات ما وراء المعرفة. (الكلمات المفتاحية: دعم تعلم مهارات ما وراء المعرفة، التعلم التعاوني، الاستدلال الرياضي، الوعي بتطبيق مهارات ما وراء المعرفة)

### Introduction

Mathematics is generally accepted as a very important school subject, and thus the teaching and learning of mathematics have been intensively studied and researched over the past ten decades. There is general agreement that learning mathematics with understanding involves more than competency in basic skills. Learning mathematics with understanding is much more than mastering arithmetic and geometry, it deals with conceptual understanding, procedural fluency, and reasoning (Kilpatrick, Swafford, & Findell, 2001). Learning mathematics with understanding is more than learning the rules and operations that students learn in school. It is about connections, seeing relationships, and knowledge reconstruction in everything that students do (Brown, Hedberg, & Harper, 1994).

According to the constructivist paradigm, students learn because they have taken prior knowledge and have reworked the new information into their current schema. A schema consists of the pieces of knowledge already present in the person. The processes that rework new information and incorporate it to prior knowledge are called assimilation and accommodation. When a new experience is incorporated into prior knowledge it is assimilated. Accommodation occurs when the new knowledge alters the knowledge, or schema (DeLay, 1996; Grabowski, 2004; Lee, Lim, & Grabowski, 2010).

Piaget (1970) believes that individuals work with independence and equality on each other's ideas, so when the students are opposed to new knowledge and interact with others they encounter something that contradicts their belief or current understanding. This is what Piaget calls "cognitive conflicts". This conflict results a case of disequilibrium. In this regard, working cooperatively may enhance students to assimilate or accommodate their knowledge and therefore reequilibrate their thinking.

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